SOLVING LOG (LN) EQUATIONS

You can only have a maximum of one log(ln) on each side of the equal sign:

Case 1: If there is a log(ln) on only one side of the equal sign:
1) Write as a single log (ln).
2) Change to exponential form and solve for x.

\[
\log_8 4 + \log_8 x = 2 \\
\log_8 4x = 2 \quad \text{write as a single log} \\
8^2 = 4x \quad \text{change to exponential form} \\
64 = 4x \quad \text{solve for x} \\
16 = x 
\]

Case 2: If there is log(ln) on both sides of the equal sign:
1) Write as a single log(ln) if necessary
2) Drop both log(ln) ... it will appear as if the log(ln) cancel out
3) Solve for x

\[
\log x + \log (x + 5) = \log 24 \\
\log x(x + 5) = \log 24 \quad \text{write as a single log} \\
x(x + 5) = 24 \quad \text{drop the log on each side} \\
x^2 + 5x - 24 = 0 \quad \text{solve for x} \\
(X + 8)(X - 3) = 0 \\
X = -8 \quad X = 3 
\]

\[X = 3 \quad \text{is the final answer because } X = -8 \quad \text{does not check.}\]

*Note: You must check answers in log(ln) problems to make sure the answer lies within the domain. The check is not shown here.

SOLVING PROBLEMS WHEN THE VARIABLE IS IN THE EXPONENT

You log(ln) both sides of an equation if and only if the variable is in the exponent (unless the equation can be factored).

1) If the variable is in the exponent, log(ln) both sides of the equation.
2) “Pop” the exponent down in front of the log(ln).
3) Solve for X (remember that \( \ln e = 1 \)).

\[
e^{3x} = 4 \\
\ln e^{3x} = \ln 4 \quad \text{In both sides of the equation} \\
3X \ln e = \ln 4 \quad \text{“pop” down the exponent} \\
3X = \ln 4 \quad \text{solve for x} \\
X = \frac{\ln 4}{3} 
\]

*Note: You do not have to check this problem because there is not a log(ln) in the original problem (although it is always a good idea to check your answers).