Trigonometry (MAC 1114) **Review Problems for Final Exam**

1. Covert each of the following degree measures to radians. Leave answers as multiple of π.
   a) 135° b) 12° c) -315°

2. Covert each of the following degree measures to degrees.
   a) $\frac{7\pi}{6}$ b) $\frac{11\pi}{3}$ a) 5 radians

3. The terminal side of angle θ in standard position goes through (-3, -4). Find the values of the six trigonometric functions of θ.

4. Draw 135° in standard position. Find a point on the terminal side and then find sin 135°, cos 135°, tan 135° without using a calculator.

5. Find all other trigonometric function values of θ given that tan $\theta = \sqrt{15}$ and θ is in quadrant III.

6. Find the reference angles for the following:
   a) 218° b) -105° c) $\frac{11\pi}{6}$

7. Use reference angles to find the exact value of each of the following **without using a calculator.**
   a) tan 150° b) sin 240° c) sec (-225°) d) sin $\frac{23\pi}{6}$

8. Find θ, if $0° < \theta < 360°$ for the following **without using a calculator.**
   a) cos $\theta = -\frac{1}{2}$ and θ in QII b) tan $\theta = \sqrt{3}$ and θ in QIII c) sin $\theta = -\frac{\sqrt{3}}{2}$ and θ in QIV

9. Use a calculator to find a value of θ between 0° and 90°. Round the answers to two decimal places. a) sin $\theta = 0.9954$ b) csc $\theta = 7.0683$ c) cot $\theta = 15.3745$

10. Find θ, if $0° < \theta < 360°$ for the following using a calculator.
    a) cos $\theta = -0.8327$ and θ in QIII b) tan $\theta = -0.6732$ and θ in QIV

11. Let ABC be a right triangle with C = 90°. If a = 29.43 cm and c = 53.58 cm, find b, A, and B.

12. From the top of a 250 feet lighthouse, the angle of depression to a ship in the ocean is 18°. How far is the ship from the base of the lighthouse?

13. a) Find the arc length if the radius is 5 c.m and the central angle is 140°.
    b) Find the area of the sector if the radius is 4.3 feet and the central angle is 200°.

14. An arc of length 3 m subtends a central angle θ in a circle of radius 12 m. Find the measure of θ in degrees and in radians.

15. The minute hand of a clock is 5.3 c.m. long. How far does the tip of the minute hand travel in 40 minutes?

16. Find the domain, range, period, and amplitude of each of the following functions.
a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$ d) $y = \csc x$ e) $y = \sec x$ f) $y = \cot x$.

17. Find the amplitude, period, and phase shift of the function, and sketch the graph of one compete period. a) $y = -\sin 3x$ b) $y = \cos\left(x - \frac{\pi}{2}\right) + 1$

18. Evaluate the exact values of the following without a calculator.
a) $\tan(\cos^{-1}(\frac{2}{7}))$ b) $\csc(\tan^{-1}(\frac{3}{4}))$

19. True or False:
a) $\sin(-\theta) = -\sin\theta$ b) $\sec(-\theta) = \sec\theta$ c) $\tan(-\theta) = \tan\theta$
d) $\cot\theta = \frac{\cos\theta}{\sin\theta}$ e) $\sin\theta = \frac{1}{\sec\theta}$ f) $\sec\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
g) $1 + \cot^2\theta = \csc^2\theta$ h) $\tan\theta = \frac{\text{adjacent}}{\text{opposite}}$

20. Fill in the blanks without using a calculator:
a) $\tan 53° = \cot$ __ b) -225° is in quadrant _____
c) cos is positive in QI and _____ d) tan is positive in QI and _____
e) $1 + \tan^2\theta =$ _____ f) $1 - \sin^2\theta =$ _____ g) $\csc\theta = \frac{1}{?}$

Verify the following identities.
21. $\frac{1+\cos x}{1-\cos x} = (\csc x + \cot x)^2$ 22. $\frac{\cos x}{1-\tan x} + \frac{\sin x}{1-\cot x} = \sin x + \cos x$

23. $\frac{\sec x + \tan x}{\sec x - \tan x} = \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x}$ 24. $\sin(180° - \theta) = \sin\theta$

25. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}\cos x$ 26. $\sin(90° + \theta) - \sin(90° - \theta) = 0$

27. $\tan\left(x + \frac{\pi}{4}\right) = \frac{1+\tan x}{1-\tan x}$ 28. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

29. Use the **Addition and/or Subtraction Formula** to find the exact value of a) $\sin 75°$ b) $\tan\frac{7\pi}{12}$

30. Use the **Double Angle Formulas** to find $\sin 2x$, $\cos 2x$, and $\tan 2x$, if $\cos x = \frac{5}{13}$ and x is in QIV.

31. Use the **Half Angle Formulas** to find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$, if $\tan x = 1$ and x is in Q III.

Solve the following equations for x if $0 \le x < 2\pi$.
32. $3\sec x + 6 = 0$ 33. $\cos x \tan x - \cos x = 0$ 34. $2\sin^2 x - 3\sin x = -1$

Solve the following equations for x if $0° \le \theta < 360°$.
35. $1 - 4\cos\theta = -2\cos^2\theta$ 36. $2\cos^2\theta + \sin\theta = 1$
37. $\sin(3\theta - 45) = -\frac{\sqrt{3}}{2}$ 38. $\cos 3\theta = -\frac{1}{2}$

Find the missing parts of each of the following triangles.

39. a = 39 cm, C = 32°, B = 110°

40. b = 100 ft, c = 60 ft, and C = 28°

41. a = 16 m, c = 7 m, B = 95°

42. a = 15 ft, b = 25 ft, c = 28 ft

Find the area of each of the following triangles:

43. a = 4, A = 40°, B = 60°

44. a = 76.3 ft, b = 109 ft, c = 98.8 ft

Eliminate the parameter t from each of the following parametric equations.

45. $x = 3 \sin t$ and $y = 4 \cos t$

46. $x = \sec t$ and $y = \tan t$

47. $x = 4 \sin t - 5$ and $y = 4 \cos t - 3$

48. $x = 5 \sin t$ and $y = -2 \sin t$

49. Write the following complex number in trigonometric form, with θ between 0 and 2π.

$4\sqrt{3} - 4i$

50. Given $z_1 = 3(\cos 60° + i \sin 60°)$ and $z_2 = 2(\cos 90° + i \sin 90°)$, find $z_1 z_2$ and z_1/z_2.

51. Find $(-2 + 2i)^{16}$ using DeMoivre's Theorem.

52. Convert the following:

a) $(-\sqrt{3}, -1)$ to polar coordinates

b) $(\sqrt{2}, -45°)$ to rectangular coordinates

53. a) Write the equation $r^2 = 4 \sin 2\theta$ with rectangular coordinates.

b) Write the equation $x^2 + y^2 = 4x$ with polar coordinates.