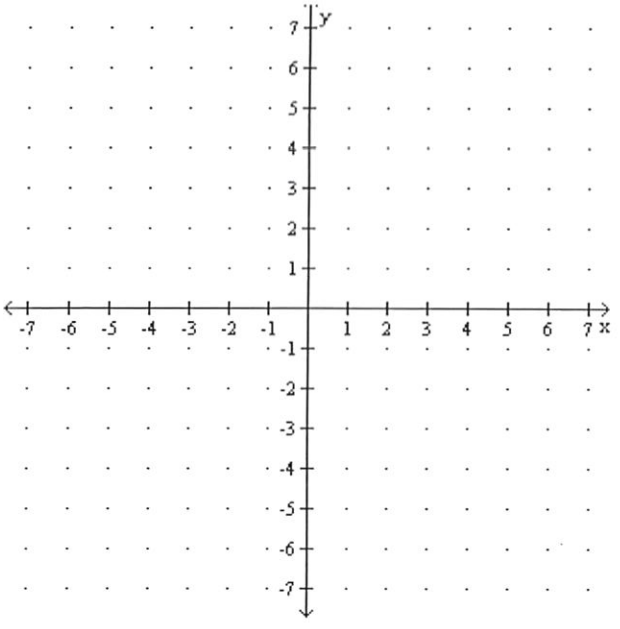
MAC 1140 *Final Exam Review Questions*

Just a reminder you must score a 60% or higher on the final exam in order to pass the course.

Graphs of Polynomials

1. **Graph** $f(x) = (x-2)^4 - 5$

Use the TABLE to plot three points



State the transformations:

_____

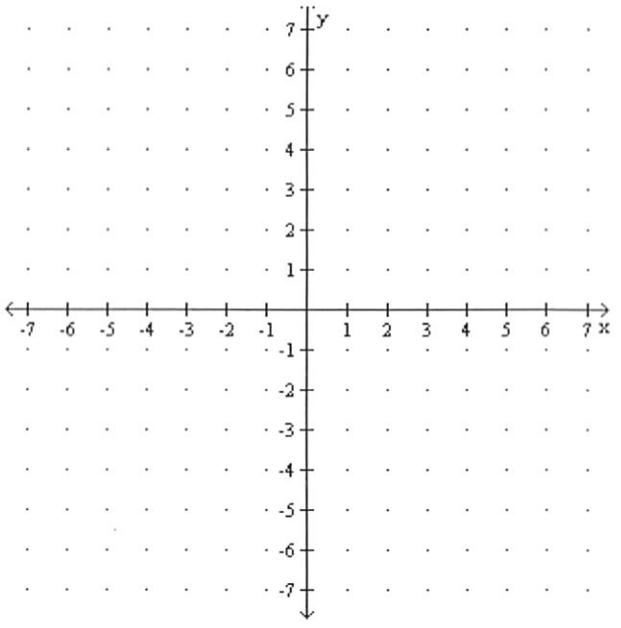
Circle the end behavior:

Rises or Falls on the left

Rises or Falls on the right

2. **Graph** $f(x) = -(x+1)^3$

Use the TABLE to plot three points



State the transformations:

_____

Circle the end behavior:

Rises or Falls on the left

Rises or Falls on the right

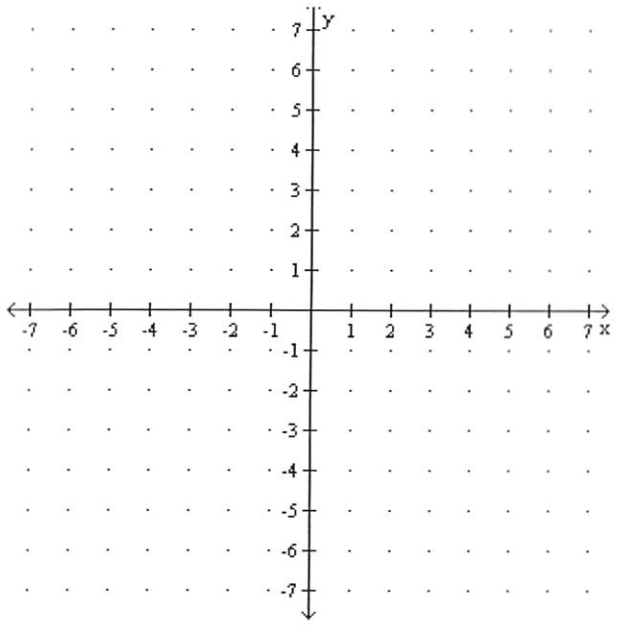
3. **Describe the end behavior of the polynomial**

$f(x) = 50x^5 - 3x^3 + 40x^2 - 20$

$f(x) \to$ _____ *as* $x \to -\infty$

$f(x) \to$ _____ *as* $x \to \infty$

4. $f(x) = x^2(x-3)$



a) x-intercepts:

_____ crosses or touches

_____ crosses or touches

b) End behavior: Use rises or falls

_____ on the left

_____ on the right

c) Maximum number of turning points: _____

d) Relative maximum of $f(x)$ is the point _____

Relative minimum of $f(x)$ is the point _____

e) Sketch a graph of $f(x)$. Use the TABLE feature to plot five points.

The Zeros of Polynomials

Find a polynomial in simplified form whose degree and real zeros are given.

5. Zeroes: –2 (multiplicity 2) and 5(multiplicity 1) degree: 3

6. Use *Descartes' Rule of Signs* to find the possible number of positive and negative real zeros for $f(x) = 3x^4 + 23x^3 + 56x^2 + 52x + 16$. Do not find any zeros.

Possible Number of positive real zeros is _____.

Possible Number of negative real zeros is _____.

7. **Explain using the *Intermediate Value Theorem* why $f(x) = x^2 - 2$ has an x-intercept between the numbers 1 and 2.**

8. **Given the function $f(x) = 6x^3 + 3x^2 - 2x + 5$, list all possible rational zeros of the function according to the *Rational Zeros Theorem*. Do not find them.**

The possible rational zeros are: _____

9. **Find the *exact values* of *all the zeros* to the function $g(x)$. Verify at least one zero using synthetic division.**

$g(x) = 6x^3 - 31x^2 + 25x + 12$

Show your synthetic division in this space:

The zeros of the function are: _____.

Write the linear factorization of g(x): _____ .

10. **Find <u>all</u> zeros and write $h(x)$ in factored form. You must verify at least <u>two zeros</u> using synthetic division. State all zeros in exact form. Do not use decimal approximations.**

$h(x) = x^4 - 4x^3 + 53x^2 - 196x + 196$

Show your synthetic division in this space:

The zeros of the function are: _____.

Write the linear factorization of $h(x)$: _____.

Functions

11. **Find the domain of each rational function. Write your answer using interval notation.**

a) $f(x) = x + 3$ b) $f(x) = \dfrac{1}{x+3}$ c) $f(x) = \sqrt{x+3}$ d) $f(x) = \dfrac{1}{\sqrt{x+3}}$

12. **Find the equation of the <u>vertical asymptote</u>(s) for each rational function.**

a) $f(x) = \dfrac{x+5}{x^2-1}$ b) $f(x) = \dfrac{3x+5}{x-1}$

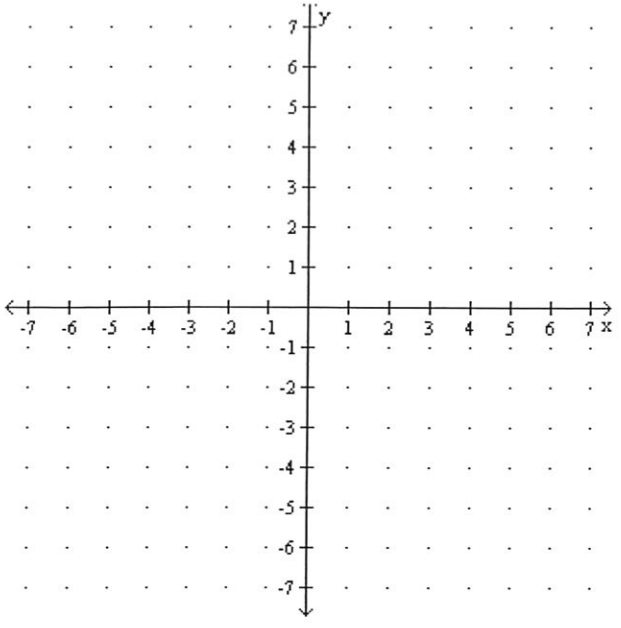
13. **Find the equation of the <u>horizontal asymptote</u> for each rational function.**

a) $f(x) = \dfrac{x+5}{x^2-1}$ b) $f(x) = \dfrac{3x+5}{x-1}$

14. **Find the equation of the <u>slant asymptote</u> for** $g(x)=\dfrac{2x^2-4x+5}{3-x}$ **using long division.**

15. **<u>Sketch a complete graph</u> of the following rational function.**
Label exact values for all *<u>intercepts</u>* and *<u>asymptotes</u>* on your graph.

$$f(x)=\frac{x+4}{x^2-4}$$



a) **The vertical asymptotes are** _____.

b) **The horizontal asymptote is** _____.

c) **The x-intercept is** _____.

d) **The y-intercept is** _____.

e) $\lim\limits_{x\to 2^-} f(x)=$ _____

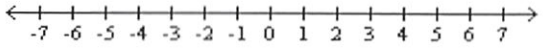
f) $\lim\limits_{x\to 2^+} f(x)=$ _____

g) $\lim\limits_{x\to \infty} f(x)=$ _____

16. **Given** $f(x)=\dfrac{x^2-4}{x-2}$. **Where is the hole? Write your answer in ordered pair form.**

17. Solve the inequality: $(x+3)(x-1)(4-x)>0$

Use the TABLE feature of the graphing calculator to determine whether the function is positive or negative in between each x-intercept.



Write the solution in interval notation: _____

18. **For the given functions f and g, find the following**

$f(x)=-2x^2-5x+7$ $\quad\quad$ $g(x)=3x^2+4x$

(a) $f(g(x))=$ $\quad\quad$ (b) $f(g(-2))=$

(c) $f(x)-g(x)=$ $\quad\quad$ (d) $\dfrac{f(x+h)-f(x)}{h}$ $\quad h\neq 0$

you will see this in calculus

19. **Find the inverse function of $f(x)=\sqrt{3x-1}$ and complete the chart.**

$f^{-1}(x)=$ _____

	Domain	*Range*
$f(x)$		
$f^{-1}(x)$		

Exponential and Logarithmic Expressions & Equations

20. **For each exponential function determine the following:**

a) $y = e^x$ **b)** $y = e^x + 2$ **c)** $y = e^{-x} - 3$

y-intercept: _____ **y-intercept:** _____ **y-intercept:** _____

$\lim_{x \to -\infty} e^x =$ _____ $\lim_{x \to -\infty} e^x + 2 =$ _____ $\lim_{x \to \infty} e^{-x} - 3 =$ _____

21. ***Expand*** **completely using the power, product or quotient rules.**

$$\ln\left(\frac{x^3\sqrt{x+1}}{(x-2)^2}\right)$$

22. **Solve.**

$\log_4 x + \log_4(x-3) = 1$

$x =$ _____

23. **Solve $5^x = 40$. Write your answer in exact form and then approximate your answer to the nearest thousandth.**

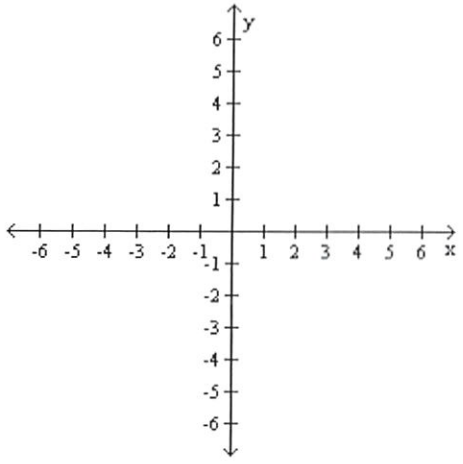
Exact answer: _____ **Approximate answer:** _____

24. **How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of interest of 7% compounded continuously.**

$t =$ _____ .

Conics

25. **Given the parabola $x^2 = 12y$ determine the following. Then, sketch the parabola using the endpoints of the focal diameter.**



Focus: _____

Directrix: _____

Endpoints of the Focal Diameter are:

_____ **and** _____

Direction of Opening: _____

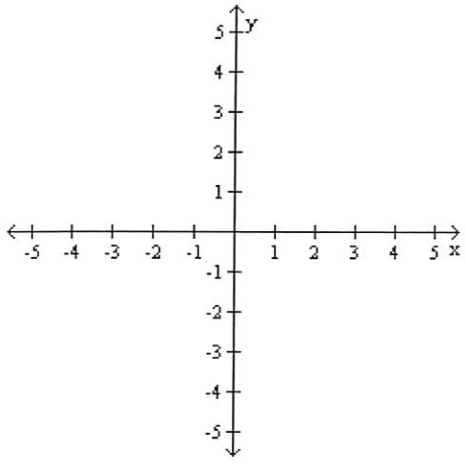
26. **Find the equation of the ellipse in standard form centered at the origin having:**

vertices : $(0,\pm 8)$
foci : $(0,\pm 4)$

The equation of the ellipse is _____.

27. **Sketch the ellipse. State exact values for the foci and the vertices of the major axis and the vertices of the minor axis.**

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$



Foci: _____

Vertices of the Major Axis are:

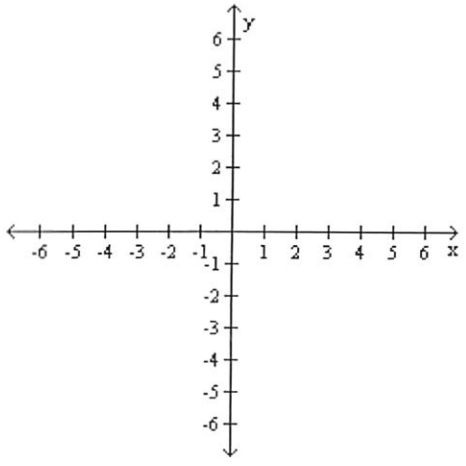
_____ **and** _____

Vertices of the Minor Axis are:

_____ **and** _____

28. **Determine the following for the hyperbola**

$$4x^2 - y^2 = 16$$



Foci: _____

Vertices are:

_____ **and** _____

Equation of the asymptotes:

_____

Systems & Matrices

29. **Solve the system by using row operations on a matrix to write the matrix in *triangular form*. Then, use *back substitution* to determine (x, y, z).**

$$x - y + z = 8$$
$$y - 12z = -15$$
$$2x + 3y - z = -2$$

a) **Write the augmented matrix for the system.**
b) **Perform row operations to obtain a matrix in triangular form.**
c) **Back substitute to obtain the solution (x, y, z).**

30. Use *Cramer's rule* to solve the system.

$$x - 3y + 7z = 13$$
$$x + y + z = 1$$
$$x - 2y + 3z = 4$$

a) Calculate *det D* by expanding across row 1. Show all of your steps!!

b) Use a calculator to determine the remaining determinants:

$D_x =$ _____

$D_y =$ _____

$D_z =$ _____

c) Determine the solutions in reduced form:

$x =$ _____ $y =$ _____ $z =$ _____

31. Find real numbers *a*, *b*, and *c* such that the graph of the function $y = ax^2 + bx + c$ contains the points (1, –5), (2, –3) and (–2, 13).

(a) Write a *matrix equation* for the system.

(b) Solve the system using an *inverse matrix*.

(c) The equation of the parabola is _____.

32. A lucky relative invested \$56,000. They invested some of the money in account A that yielded 3% simple interest, some of the money in account B that yielded 3.5% simple interest, and the remainder in account C that yielded 4% simple interest. Your relative said the total interest earned from the three investments for one year was \$2020. Your relative also said the amount invested at 4% was twice the amount invested at 3%. Can you determine how much your relative invested in each of the three accounts?

Write your system here: Write your augmented matrix here:

Amount invested @ 3% _____

Amount invested @ 3.5% _____

Amount invested @ 4% _____

Be able to do questions 33 - 35 by hand. You will not be allowed to use a calculator on this part of the final.

33. Given the matrix $A = \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix}$

(a) The determinant of A = _____.

(b) Use the quick method to find A^{-1}.

34. Find AB. Show all of your work.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 3 \\ 7 & 1 \\ 5 & -2 \end{bmatrix}$$

35. **Use the given matrices to determine the following by hand:**

$$A = \begin{bmatrix} -9 & 1 \\ 3 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & -11 \\ -3 & 6 \end{bmatrix}$$

a) **Find $2A + B$**

b) **Find $A - B$**

c) **Find AB**

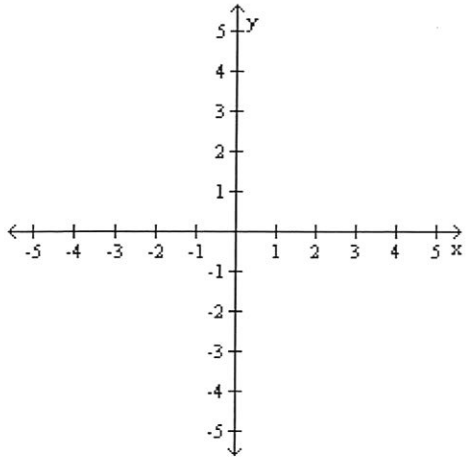
d) **Find BA**

36. **Solve and check your solutions.**

$$x^2 + y^2 = 100$$
$$y = x + 2$$

37. **Graph the system of inequalities:**

$$x^2 + y^2 < 9$$
$$y > 0$$



Sequences & Series

38. **Write down the first five terms of the recursive sequence.**

$a_1 = -3 \qquad a_n = 2a_{n-1} + 1$

The first five terms of the sequence are:

_____, _____, _____, _____, _____

39. **The given pattern continues. Write down a formula for the nth term of the sequence suggested by the pattern.**

$5, \frac{6}{4}, \frac{7}{9}, \frac{8}{16}, \frac{9}{25}, \ldots$

$a_n =$ _____.

40. **Find the *sum* of the sequence.**

$$\sum_{k=0}^{4} (k^2 - 4)$$

The sum of the sequence is _____.

41. **Given the sequence:**

4, 9, 14, 19, . . .

a) **Find the n^{th} term.**

b) **Find the 50^{th} term of the sequence.**

c) **Find the *sum* of the first 50 terms of the sequence.**

42. **Given the sequence**

2, 6, 18, 54, . . .

a) **Find a formula for the n^{th} term:** _____

b) **The 13^{th} term is** _____.

c) **The sum of the first 13 terms of the sequence is** _____.

43. **Determine the *sum* of the infinite geometric series.**

$1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + ...$

$S_\infty =$ _____

44. **Your starting salary is $38,000 a year and every year you receive a $3000 raise**

a) **Determine your salary at the end of your 7^{th} year.**

b) **Determine the total amount of money you made over the 7-year period.**

45. **A bacteria culture initially has 2000 bacteria and its size increases by 4% every hour. How many bacteria are there after 10 hours? In other words, how many bacteria are there in the 11^{th} term of the sequence?**

46. **Given the sequence: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60 and 65**

a) **Find the sum of the sequence using your calculator.**
b) **Find the sum of the sequence using the method of Gauss.**
c) **Find the sum of the sequence using the formula for the sum of an arithmetic sequence.**
d) **Is the method of Gauss actually the same as the formula for the sum of an arithmetic sequence? If so, explain.**
e) **Does the method of Gauss work for geometric sequences? Explain using an example.**

47. **Expand** $(2x + y)^6$

Determine whether the given quadratic function (for # 48 and 49) has a maximum value or minimum value, and then find the value.

48. $f(x) = -4x^2 - 6x + 2$

49. $f(x) = 3x^2 - 8x + 2$

Solve each inequality algebraically:

50. $\dfrac{x(x^2+1)(x-2)}{(x-1)(x+1)} \geq 0$

51. $\dfrac{(2-x)^3(3x-2)}{x^3+1} < 0$

52. Find the inverse of the following matrix: $\begin{bmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$

53. Graph the following system of linear inequalities: $\begin{cases} x \geq 0 & y \geq 0 \\ x + y \geq 2 \\ x + y \leq 8 \\ x + 2y \geq 1 \end{cases}$

54. Expand the following expression using the Binomial Theorem: $\left(\sqrt{x} + \sqrt{2}\right)^6$

55. Determine whether the following infinite geometric series converges or diverges. If it converges, find its sum: $\sum_{k=1}^{\infty} 3\left(\frac{2}{3}\right)^k$

56. Use the given zero to find the remaining zeros of the following polynomial function:

$g(x) = 3x^5 + 2x^4 + 15x^3 + 10x^2 - 528x - 352; zero: -4i$

57. Find the partial fraction decomposition of the following rational expression: $\dfrac{x^2+1}{x^3+x^2-5x+3}$