I. Exact and Approximate Numbers

**Exact number**
1. Any number obtained by a counting process. Example: 60 students in a class — 60 is an exact number
2. Any number given by definition. Example: One hour equals 60 minutes — 60 is an exact number

**Approximate Number**
1. Any number obtained by a process of measurement
   Example: The distance between two cities is 60 miles — 60 is an approximate number. The distance may be a little more or a little less than 60 miles.

II. Accuracy and Precision

**Accuracy** of an approximate number is judged by the number of significant digits. Example: 4.63 cm is more accurate than 4.6 cm.

**Precision** of an approximate number refers to the decimal position of the significant digit furthest to the right. Example: 26.53 is more precise than 2512 (26.53 was measured to hundredths, 2512 to units)

III. Significant Digits

are those digits that have meaning relative to the measuring process used to obtain them.

Five rules governing which digits of an approximate number are significant:

1. **Nonzero digits** are always significant
2. **Zeros that are preceded and followed by significant digits** are always significant.
3. **Final Zeros to the right of a decimal point** are significant
4. **Final zeros on a whole number** are not significant (unless further information indicates otherwise).
5. **If no digits left of the decimal point**, zeros between the decimal point and the first digit are not significant.

Examples
1. 14.24 has four significant digits (Rule 1)
2. .0036 has two significant digits (Rule 1 & 5)
3. 14.0024 has six significant digits (Rule 3)
4. 7001 has four significant digits (Rule 2)
5. 8.9000 has five significant digits (Rule 3)
6. 0.03600 has four significant digits (Rule 1 & 5)
7. In the statement, “Hawaii is 2400 miles west of Los Angeles,” there are two significant figures (Rule 4).
8. If the distance to the nearest mile between cities is 20 miles, there are two significant figures (Rule 4).

IV. Rounding to Indicated Place Value

1. If the digit to the right of the indicated place value is less than 5 (0, 1, 2, 3, 4), retain the digit at place value.
2. If the digit to the right of the indicated place value is greater than or equal to 5 (5, 6, 7, 8, 9), add 1 to the digit.
3. Eliminate all digits to the right of the indicated place value.

Examples:
1. Round 2.57834 to the nearest hundredth
   i. 7 is hundredth digit
   ii. 8 is first digit to the right
   iii. ANSWER: 2.58 (Rule 2)
2. Round 37.50 to nearest unit
   i. 7 is unit digit
   ii. 5 is first digit to the right
   iii. ANSWER: 38 (Rule 2)
3. Round 0.0031 to the nearest thousandth
   i. 3 is thousandth digit
   ii. 1 is first digit to the right
   iii. ANSWER: 0.003 (Rule 1)
V. Rounding in Calculations

Adding or Subtracting approximate numbers: Express result with the precision of the least precise number

Example: add 73.2, 8.0627, 93.57, 66.296
73.2 is least precise (precise to tenths place)
241.1286 is total
ANSWER 241.1

Example: subtract 36.1 from 727.842
36.1 is less precise (precise to tenths place)
691.742 is result of subtraction
ANSWER: 691.7

Multiplying or Dividing approximate numbers: Express result to the accuracy of the least accurate number

Example: divide 292.6 by 3.4
3.4 is least accurate (only two significant digits)
86.05824 is result of division
ANSWER: 86 (two significant digits)

Example: multiply 2.4832 by 30.5
30.5 is less accurate (only three significant digits)
75.7315 is result of multiplication
ANSWER: 75.7 (three significant digits)

Taking the root of an approximate number: Express result to the accuracy of the number

Example: take \( \sqrt{3.7} \) (two significant digits)
Result is 1.9235384
ANSWER: 1.9 (two significant digits)

Rounding before and after calculations:
Before calculation, all numbers except the least precise or least accurate may be rounded to one place beyond that of the least precise or least accurate

Example: rounding before calculation

<table>
<thead>
<tr>
<th>Number</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>73.2</td>
<td>tenths</td>
</tr>
<tr>
<td>8.0627</td>
<td>hundredths</td>
</tr>
<tr>
<td>93.57</td>
<td>hundredths</td>
</tr>
<tr>
<td>66.296</td>
<td>hundredths</td>
</tr>
<tr>
<td>241.1287</td>
<td></td>
</tr>
</tbody>
</table>

ranging after calculation: ANSWER: 241.1

Note: if calculator is used, rounding as shown above need not be followed.

Exact number in a calculation: If an exact number is included in a calculation, there is no limit to the number of decimal places it may take. The accuracy of the result is limited only by the approximate numbers involved.

Example: consider the exact number 144 and the approximate number 2.7.

If these are added or subtracted, express result in tenths:

\[ 144 + 2.7 = 146.7 \]
\[ 144 - 2.7 = 141.3 \]

If these are multiplied or divided, express result to two significant digits:

\[ 144 \times 2.7 = 390 \]
\[ 144 / 2.7 = 53 \]